POOL FILM BOILING HEAT TRANSFER FROM A HORIZONTAL CYLINDER TO SATURATED LIQUIDS

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Abstract—Referring to free convective film boiling heat transfer from a horizontal cylinder, many kinds of correlations of heat transfer have been proposed. However, none of them correlates the available data for a wide range of kind of the boiling liquid, the diameter of the cylinder, the degree of superheat and the system pressure.

The integral method of boundary layer was applied to solve the equation of momentum and that of energy for this problem, taking into account of the inertia force in the former and the convective term in the latter and two nondimensional parameters were derived from this analysis. By employing these parameters, a semi-theoretical correlating equation accurate enough for practice over a wide range of parameters was proposed.

a, thermal diffusivity; q , heat flux; R , constant, equation (15); q , heat flux; R , constant, equation (19); R , heat flux; R , latent heat of vaporization; R , radius of cylinder; R ,		NOMENCLATURE	Pr_F^* ,	modified Prandtl number by
B, constant, equation (19); r_v , latent heat of vaporization; Cp , specific heat at constant pressure; R , radius of cylinder; D , diameter of cylinder; Sp , dimensionless superheating, $Sp \equiv Cp_v\Delta T_{sav}/r_vPr_v$; Sp , acceleration due to gravity; Sp , temperature profile, equation (5); Sp , saturation temperature; Sp , $Sp \equiv Cp_v\Delta T_{sav}/r_vPr_v$; Sp ,	a,	thermal diffusivity;	_	Frederking, equation (36);
Cp ,specific heat at constant pressure; R ,radius of cylinder; D .diameter of cylinder; Sp ,dimensionless superheating, $F(\eta)$.velocity profile, equation (4); $Sp \equiv Cp_v\Delta T_{sav}/r_vPr_v$; g .acceleration due to gravity; T .temperature; $G(\eta)$.temperature profile, equation (26); T_s .saturation temperature; Gr_D .Grashof number, equation (26); ΔT_{sav} .temperature difference between heated wall and saturated liquid; $I(\varphi)$.symbol of a function, equation (27); u_{φ} ,tangential component of velocity; I .numerical constants, equation (29); u_{φ} ,representative velocity, equation (4); k_1, k_2, k_3 .numerical constants, Table 2; X .dimensionless number, equation (34); k_1, k_2, k_3 .numerical constants, equation (25); y .radial co-ordinate; Nu_D .average Nusselt number, equation (28); y .radial co-ordinate; Nu_D .average Nusselt number, equation (28); y .dimensionless number, equation (35). P .system pressure; P .	A,	constant, equation (15);	q,	
D. diameter of cylinder; Sp , dimensionless superheating, $Sp \equiv Cp_v\Delta T_{\rm sal}/r_vPr_v$; g . acceleration due to gravity; T , temperature; $G(\eta)$, temperature profile, equation (26); $\Delta T_{\rm sal}$, temperature difference between $I(\varphi)$. symbol of a function, equation (27); u_{φ} , tangential component of velocity; I , numerical constants, equation (29); I , numerical constants, Table 2; I ,	B ,	constant, equation (19);	r_v ,	latent heat of vaporization;
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g. acceleration due to gravity; T. temperature; $G(\eta)$. temperature profile, equation (5); T_s , saturation temperature; Gr_D . Grashof number, equation (26); $\Delta T_{\rm sat}$, temperature difference between $I(\varphi)$, symbol of a function, equation (27); u_{φ} , tangential component of velocity; u_{φ} , tangential component of velocity; numerical constants, equation (29); u_{φ} , representative velocity, equation (29); u_{φ} , dimensionless number, equation (34); u_{φ} , radial co-ordinate; u_{φ} , r	D,	diameter of cylinder;	Sp,	dimensionless superheating,
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Gr_D . Grashof number, equation (26); $\Delta T_{\rm sat}$, temperature difference between $I(\varphi)$, symbol of a function, equation (27); u_{φ} , tangential component of velocity; I , numerical constants, equation (29); I ,	g.	acceleration due to gravity;	<i>T</i> ,	temperature;
$I(\varphi), \qquad \text{symbol of a function, equation} \\ (27); \qquad \qquad u_{\varphi}, \qquad \text{tangential component of velocity;} \\ I. \qquad \text{numerical constants, equation} \\ (29); \qquad \qquad (4); \\ k_1, k_2, k_3, \qquad \text{numerical constants, Table 2;} \qquad X, \qquad \text{dimensionless number, equation} \\ Nu_D(\varphi), \qquad \text{local Nusselt number, equation} \\ (25); \qquad y, \qquad \text{radial co-ordinate;} \\ Nu_D, \qquad \text{average Nusselt number, equation} \\ (28); \qquad y, \qquad \text{radial co-ordinate;} \\ Pr, \qquad \text{Prandtl number;} \qquad \text{Greek symbols} \\ Pr^*, \qquad \text{modified Prandtl number, equa-} \qquad \beta_1, \beta_2, \beta_3, \text{numerical constants, equations} \\ \beta_1, \beta_2, \beta_3, \text{numerical constants, equations} \\ \end{cases}$	$G(\eta)$.	temperature profile, equation (5);		saturation temperature;
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Gr_{D} ,	Grashof number, equation (26);	$\Delta T_{\rm sat}$,	temperature difference between
I, numerical constants, equation (29); (4); I ,	$I(\varphi)$,	symbol of a function, equation		heated wall and saturated liquid;
$(29); \qquad (4); \qquad$	_	(27);	u_{φ} ,	tangential component of velocity;
k_1, k_2, k_3 , numerical constants, Table 2; X , dimensionless number, equation $Nu_D(\varphi)$, local Nusselt number, equation (25); y , radial co-ordinate; $Nu_D(\varphi)$, average Nusselt number, equation (28); y , dimensionless number, equation (25). P , system pressure; Pr , Pr Prandtl number; Pr *, Pr Prandtl number, equation Pr *, Pr Prandtl number, equation Pr *,	I,	1	u_0 ,	representative velocity, equation
$Nu_D(\varphi)$, local Nusselt number, equation (25); y , radial co-ordinate; Nu_D , average Nusselt number, equation (28); y , dimensionless number, equation (28); (35). P , system pressure; Pr , Prandtl number; Pr^* , modified Prandtl number, equa- Pr^* , modified Prandtl number, equa-				(4);
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(28); (35). P. system pressure; Pr, Prandtl number; Greek symbols Pr^* , modified Prandtl number, equa- $\beta_1, \beta_2, \beta_3$, numerical constants, equations			у,	radial co-ordinate;
P. system pressure; Pr. Prandtl number; Pr*, Greek symbols $\beta_1, \beta_2, \beta_3, \text{ numerical constants, equations}$	Nu_{D} ,		Y ,	dimensionless number, equation
Pr, Prandtl number; Greek symbols modified Prandtl number, equa- $\beta_1, \beta_2, \beta_3, \text{ numerical constants, equations}$	_			(35).
Pr^* , modified Prandtl number, equa- $\beta_1, \beta_2, \beta_3$, numerical constants, equations		•		
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tion (23); (8)-(10);	Pr*,		$\beta_1, \beta_2, \beta_3$	numerical constants, equations
		tion (23);		(8)-(10);

 $\gamma_1, \gamma_2, \gamma_3$, numerical constants, equations (11)–(13);

 δ , thickness of vapour film;

η, dimensionless radial coordinate, equation (3);

equation (3),

 λ . thermal conductivity;

 μ , viscosity;

v, kinematic viscosity;

 ρ , density;

 φ , angular position measured from

the bottom of the cylinder;

 ψ , symbol of a function, equation (18).

Subscripts

W, heated surface wall;
P, vapour-liquid interface;

l, liquid;v, vapour.

1. INTRODUCTION

As a HEATED surface is covered with vapour in film boiling, this vapour layer becomes the principal thermal resistance for heat flow. Therefore, it is the most important subject in the investigation of film boiling heat transfer to understand the heat transfer mechanism through this vapour layer. Theoretical treatments of film boiling heat transfer proposed by many investigators have been based on a model of thin vapour film which develops along the heated surface and has the characteristics of boundary layer.

Since Bromley [1] proposed a formal derivation of an equation for film boiling in 1950, several correlations of heat transfer from horizontal cylinder have been published. For example, to make the Bromley's equation applicable to wide range of diameter of cylinder, Breen and Westwater [2] suggested a correlating equation which included the critical wave length of Taylor instability at the vapour–liquid interface as a parameter. But this equation fails to correlate some experimental results for carbon dioxide [5] and helium [3] from thin wire.

On the other hand, Frederking [3] analysed the film boiling heat transfer by the integral method which had been developed by Krwzhilin [4] for condensing vapour on a vertical plate and a horizontal cylinder. Krwzhilin took account for the inertia force in the momentum equation and the convective effect in the energy equation. Frederking derived some possible dimensionless parameters of heat transfer and correlated some available experimental results at that time. And it was shown that a good correlation of data was obtained by the above parameters for film boiling of helium, oxygen and nitrogen. However, it seems that there is no logical inevitability in the process of composing one of the dimensionless parameters. By the way authors themselves plotted the experimental results over a wide range of parameters with Frederking's dimensionless parameters, obtaining a favourable correlation. Therefore, it is reasonable to conceive that the Frederking's correlation touches the core of the essential mechanism of film boiling heat transfer.

In this study, by using the similar integral method that Frederking applied, an analysis was developed to solve the fundamental equations of momentum and energy transportation, in which the velocity and the temperature profiles were postulated to be of arbitrary functional forms. After the equations were solved, several sets of concrete functional forms were assigned to the velocity and temperature profiles, and the corresponding equations of correlation were obtained. Then, comparing the equations with available data of film boiling heat transfer in the region where coincidence should be expected, a set of nondimensional parameters to be used in correlation was selected.

The dimensionless parameters thus derived were calculated for all the available experimental data. Though the data have covered a wide range of parameters, a fairly good correlation was obtained.

Finally, an empirical equation which can stand for the data points at best was determined

and thus a semi-theoretical correlating equation of film boiling heat transfer from a horizontal cylinder to saturated liquid was proposed.

2. ANALYSIS

Film boiling from a horizontal cylinder is characterized by the existence of a vapour film surrounding the heated surface. Bubbles depart from the top part of the cylinder. A heated cylinder which has a radius, R, and whose surface is maintained at a uniform temperature, T_w , is submerged horizontally in a stagnant saturated liquid at a temperature, T_s , as shown in Fig. 1. Vapour film with thickness, $\delta(\varphi)$, develops along the cylinder surface. φ is an angular co-ordinate measured from the bottom of the cylinder.

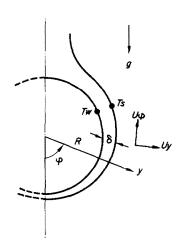


Fig. 1. Physical model and co-ordinates,

A few assumptions are made for the derivation of the integrated fundamental equations of boundary layer; (1) the vapour-liquid interface is smooth, (2) the vapour film has the nature of boundary layer and (3) the physical properties of vapour are independent of temperature.

The equations of momentum and energy transportation can be represented as follows,

referring to Fig. 1

$$-\frac{1}{R}\frac{\mathrm{d}}{\mathrm{d}\varphi}\int_{0}^{\delta}\rho_{v}u_{\varphi}^{2}\,\mathrm{d}y + g\sin\varphi\int_{0}^{\delta}(\rho_{I}-\rho_{v})\,\mathrm{d}y$$

$$-\mu_{v}\left(\frac{\partial u_{\varphi}}{\partial y}\right)_{W} + \mu_{v}\left(\frac{\partial u_{\varphi}}{\partial y}\right)_{P} = 0 \qquad (1)$$

$$-\lambda_{v}\left(\frac{\partial T}{\partial y}\right)_{W} = \frac{1}{R}\frac{\mathrm{d}}{\mathrm{d}\varphi}\int_{0}^{\delta}\rho_{v}\,Cp_{v}u_{\varphi}(T-T_{s})\,\mathrm{d}y$$

$$+\frac{r_{v}}{R}\frac{\mathrm{d}}{\mathrm{d}\varphi}\int_{0}^{\delta}\rho_{v}\,u_{\varphi}\,\mathrm{d}y = 0. \qquad (2)$$

Reducing the co-ordinate, the velocity and the temperature to non-dimensionsional forms, we have

$$\eta \equiv \frac{y}{\delta}, \qquad F(\eta) \equiv \frac{u_{\varphi}}{u_0}, \qquad G(\eta) \equiv \frac{T - T_s}{T_w - T_s}.$$
(3), (4), (5)

Then equations (1) and (2) become as follows.

$$\beta_{1} \frac{1}{R} \frac{d}{d\varphi} (u_{0}^{2} \delta) = g \frac{\rho_{1} - \rho_{v}}{\rho_{v}} \delta \sin \varphi$$

$$- v_{v} (\gamma_{1} - \gamma_{2}) \frac{u_{0}}{\delta} \qquad (6)$$

$$\left(\beta_{3} \Delta T_{\text{sat}} + \beta_{2} \frac{r_{v}}{C p_{v}}\right) \frac{1}{R} \frac{d}{d\varphi} (u_{0} \delta)$$

$$= - \gamma_{3} \Delta T_{\text{sat}} \frac{a}{\delta} \qquad (7)$$

where β_1 , β_2 , β_3 , γ_1 , γ_2 and γ_3 are all numerical constants characteristic of the dimensionless velocity and temperature profiles, namely $F(\eta)$ and $G(\eta)$. They are defined by the following identities,

$$\beta_1 \equiv \int_0^1 \{F(\eta)\}^2 d\eta, \qquad \beta_2 \equiv \int_0^1 F(\eta) d\eta$$
$$\beta_3 \equiv \int_0^1 F(\eta) G(\eta) d\eta \qquad (8), (9), (10)$$
$$\gamma_1 \equiv F'(0), \qquad \gamma_2 \equiv F'(1), \qquad \gamma_3 \equiv G'(0).$$

(11), (12), (13)

Integrating the equation (7) quite formally and then substituting it into the equation (6). we obtain

$$\frac{\beta_1}{R} A^2 \frac{1}{\delta^2} \left\{ 2 \int_0^{\varphi} \frac{d\varphi}{\delta} - \left(\int_0^{\varphi} \frac{d\varphi}{\delta} \right)^2 \frac{d\delta}{\delta \varphi} \right\}
+ v_v (\gamma_1 - \gamma_2) \frac{A}{\delta^2} \int_0^{\varphi} \frac{d\varphi}{\delta} = \frac{g(\rho_1 - \rho_v)}{\rho_v} \delta \sin \varphi \quad (14)$$

$$A \equiv -\frac{\gamma_3 a R}{\beta_3 \left[1 + (\beta_2/\beta_3)(r_v/Cp_v \Delta T_{\text{sat}})\right]}.$$
 (15)

Assuming that

$$2 \gg \frac{\mathrm{d}\delta}{\mathrm{d}\varphi} \int_{0}^{\varphi} \frac{\mathrm{d}\varphi}{\delta} \tag{16}$$

the equation (14) becomes

$$\delta^{3}\psi = B \int_{0}^{\varphi} \frac{d\varphi}{\delta}, \qquad \psi \equiv \sin \varphi \quad (17), (18) \qquad \frac{1}{Nu_{D}} \equiv \frac{1}{\pi} \int_{0}^{\varphi} Nu(\varphi) d\varphi$$

$$B \equiv \frac{\rho_v}{g(\rho_I - \rho_v)} \left\{ \frac{2\beta_1 A^2}{R} + v_v (\gamma_1 - \gamma_2) A \right\}. \tag{19}$$

Differentiation of the equation (17) with respect to φ gives,

$$3\delta^3\psi\delta' + \delta^4\psi' - B = 0. \tag{20}$$

The solution is obtained easily and the vapour film thickness can be calculated,

$$\delta = \left(\frac{4 \int_{0}^{\varphi} (\sin \varphi)^{\frac{1}{3}} d\varphi}{3 (\sin \varphi)^{\frac{4}{3}}}\right)^{\frac{1}{4}} B^{\frac{1}{4}}.$$
 (21)

B can be determined by the equations (15) and

$$B = \frac{\rho_v v_v^2 D}{2g(\rho_l - \rho_r)} \left\{ \frac{2\beta_1 \gamma_3^2 - \beta_3 \gamma_3 (\gamma_1 - \gamma_2) Pr^*}{\beta_3^2 Pr^{*2}} \right\}$$
(22)

$$Pr^* \equiv Pr \left(1 + \frac{\beta_2}{\beta_3} \frac{r_v}{Cp_v \Delta T_{ext}} \right).$$
 (23)

The heat flux can be calculated as follows.

$$q = -\lambda_r \left(\frac{\partial T}{\partial y}\right)_{\mathbf{w}} = -\lambda_r \frac{\Delta T_{\text{sat}}}{\delta} G'(0)$$
$$= -\frac{\gamma_3 \lambda_r \Delta T_{\text{sat}}}{\delta}. \tag{24}$$

Therefore the local Nusselt number, $Nu_{p}(\varphi)$, and the average Nusselt number. \overline{Nu}_D , become as follows,

(14)
$$Nu_{D}(\varphi) \equiv \frac{qD}{\lambda_{v} \Delta T_{\text{sat}}} = \frac{\gamma_{3}D}{\delta}$$
(15)
$$= -2^{\frac{1}{4}} \gamma_{3} \frac{1}{I(\varphi)} \left[-\frac{\beta_{3}}{\gamma_{3}(\gamma_{1} - \gamma_{2})} \right]^{\frac{1}{4}}$$

$$\times \left[Gr_{D} \frac{Pr^{*2}}{Pr^{*} - \left[2\gamma_{3}\beta_{1}/\beta_{3}(\gamma_{1} - \gamma_{2}) \right]} \right]^{\frac{1}{4}}$$
(25)
(16)
$$Gr_{D} \equiv \frac{D^{3}g\rho_{v}(\rho_{1} - \rho_{v})}{\mu_{v}^{2}}.$$

$$I(\varphi) \equiv \left(\frac{4\int_{0}^{\varphi} (\sin\varphi)^{\frac{1}{4}} d\varphi}{3\left(\sin\varphi\right)^{\frac{1}{4}}} \right)^{\frac{1}{4}}$$
(26), (27)
$$\overline{Nu}_{D} \equiv \frac{1}{\pi} \int_{0}^{\varphi} Nu(\varphi) d\varphi$$
(19)
$$= -2^{\frac{1}{4}} \gamma_{3} \frac{1}{\overline{I}} \left[-\frac{\beta_{3}}{\gamma_{3}(\gamma_{1} - \gamma_{2})} \right]^{\frac{1}{4}}$$

$$\text{spect} \qquad \times \left[Gr_{D} \frac{Pr^{*2}}{Pr^{*} - \left[2\gamma_{2}\beta_{1}/\beta_{2}(\gamma_{1} - \gamma_{2}) \right]} \right]^{\frac{1}{4}}$$
(28)

$$\frac{1}{I} = \frac{1}{\pi} \int_{0}^{\pi} \frac{d\varphi}{I(\varphi)} = 0.806.$$
 (29)

(28)

3. CORRELATION OF FILM BOILING HEAT TRANSFER

3.1 Selection of the dimensionless parameter for correlation

If the actual functional forms of $F(\eta)$ and $G(\eta)$ are assumed, $\beta_1, \beta_2, \beta_3, \gamma_1, \gamma_2$ and γ_3 in the equation (28) can be calculated by the definitions (8)–(13). At this point two velocity and two temperature profiles will be postulated. Then four possible combinations of velocity and temperature profile result. They are listed in Table 1.

Table 1. Combinations of velocity and temperature profile

$G(\eta)$ $F(\eta)$	$\eta - \eta^2$	$2\eta - \eta^2$
$\frac{1-\eta}{(1-\eta)^2}$	case ① case ③	case ② case ④

The velocity profile of $(\eta - \eta^2)$ corresponds to zero vapour velocity at the vapour-liquid interface, namely stagnant liquid, and that of $(2\eta - \eta^2)$ to zero shearing stress at the interface, namely freely movable liquid with vapour. On the other hand, the temperature distribution of $(1 - \eta)$ means that the heat transfer from the cylinder is due to only heat conduction through the vapour film. When the numerical constant was calculated by definitions (8)–(13) and was substituted into the equation (28) for all cases from (1) to (4), the correlating equations given in Table 2 are obtained.

Now above four equations are compared with experimental results to select the appropriate dimensionless parameters. Authors' data [5] of film boiling from horizontal cylinder of 13 mm dia. to saturated water at atmospheric pressure are chosen for comparison because (1) diameter of the cylinder is suitable for the assumed laminar boundary-layer treatment, (2)

Table 2. Examples of the equation (28)

C	Numerical constants			
Case	k_1	k ₂	k ₃	
(I)	0.433	0.400	2:00	
(2)	0.567	2.13	2:67	
(3)	0.641	1.33	3.33	
<u>(4)</u>	0.819	8.00	5:00	

$$\overline{Nu}_{D} = k_{1} \left(Gr_{D} \frac{Pr^{*2}}{Pr^{*} + k_{2}} \right)^{\frac{1}{2}}$$

$$Pr^{*} \equiv Pr \left(1 + k_{3} \frac{r_{o}}{Cp_{v} \Delta T_{sat}} \right)$$

temperature difference between the heated surface and saturated liquid is comparatively small. and (3) the vapour film is considered to be thin for reason of large latent heat. The result of comparison of four equations with the typical data of water is shown in Fig. 2, a non-dimensional superheating $Sp \equiv Cp_v \Delta T_{\rm sav}/r_v Pr_v$ being the abscissa and a modified Nusselt number $\overline{Nu}_D/(Gr_D Pr)^{\frac{1}{2}}$ the ordinate.

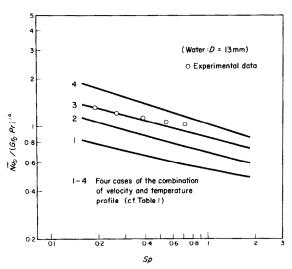


Fig. 2. Comparison of analytical results with experimental data (water, 1 ata).

Considering the physical situation at the vapour-liquid interface, the zero vapour velocity condition seems to be more likely than the zero shearing stress condition, except for near critical liquids. In Fig. 2, experimental results fit the case 3 calculation best where the zero vapour velocity condition is postulated. The correlating equation of case 3 can be represented as follows,

$$\overline{Nu}_{D} = 0.641 \left[Gr_{D} \frac{Pr^{*2}}{Pr^{*} + 1.33} \right]^{\frac{1}{4}}$$
 (30)

$$Pr^* \equiv Pr \left[1 + 3.33 \frac{r_v}{Cp_v \Delta T_{\text{sat}}} \right]^{\frac{1}{4}}$$
 (31)

which will be referred to as a basis of discussion hereafter.

3.2 Comparison with experimental results

Figure 3 is a plot of the typical authors' data [5] in which the average Nusselt number \overline{Nu}_{D} is taken as the ordinate and the dimensionless parameter $Gr_{D} \cdot Pr^{*2}/(Pr^{*} + 1.33)$ appearing in the right-hand side of the equation (30) as the abscissa. Properties of vapour were evaluated at the film temperature, $(T_w + T_s)/2$. Paying attention to a kind of boiling liquid in the figure, a large temperature difference corresponds to a small value of abscissa, and a large diameter of cylinder to a large value of abscissa. The data of water and ethyl alcohol show good agreement with the analytical result of the equation (30). The Nusselt number of n-pentane, however, is in slight excess of the analysis for a fixed value of abscissa. The latent heat of pentane is much smaller than that of water and a relatively large amount of vapour might have been generated, resulting in an enhanced degree of turbulence or disturbance in the vapour film, accordingly giving rise to a different mechanism of heat transfer from that of laminar boundary layer.

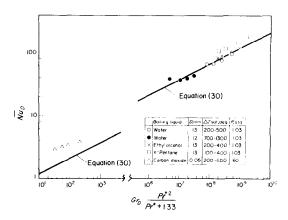


Fig. 3. Comparison of the equation (30) with some typical data.

This effect of increased film thickness causes the deviation from the analytical result, as seen in the data of boiling in water from a cylinder of 12 mm dia. In the case of boiling of carbon dioxide from a thin wire, there is a considerable deviation from the analysis. The reason for this deviation may be as follows. As

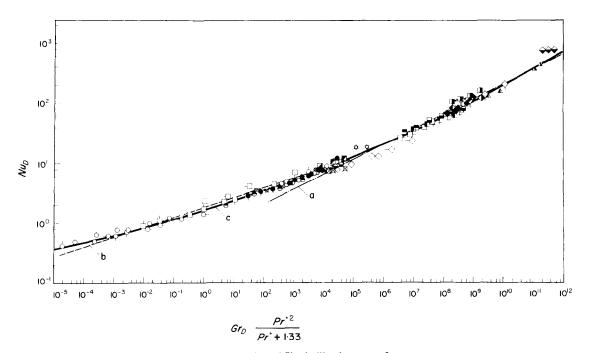


Fig. 4. Correlation of film boiling heat transfer.

Table 3. The keys in Fig. 4 and the related conditions of experiments

Keys	Boiling liquid	Pressure	Reduced pressure	Cylinder diameter	Temperature difference between the heated surface and the satulated liquid	Heat-transfer coefficient	References
		(ata)		(mm)	(deg)	(kcal/m²hdeg)	
6		1.00	0.43	0.0055	100-600	6009-10000	[3] [3]
\sim	Helium	1.00	0.43	0.013	100-600	3000-5000	[3]
Ž		1.00	0.43	0.0215	100-600	2500-4000	[3]
, (-		1.00	0.43	0.051	100–600	1000–2000	[3]
		1.00	0.029	0.010	100-600	2500	[3]
\Box	Nitrogen	1 00	0.029	0.031	100-600	1200-1500	[3]
\Box	Millogen	1.00	0.029	0.100	100-800	650-800	[3]
\Box		1.03	0.030	8.9	200–800	100	[1]
৾		1.03	0.020	0.635	100-300	240	[6]
\rightarrow	Oxygen	1.03	0.020	1.75	100-300	150-1170	[6]
· 👇		1.03	0.020	19.1	100-300	110-130	[6]
OQQQ 40000000000000000000000000000000000		75	1.0	0.2	100-800	900-1000	[5]
		60	0.80	0.05	250-650	1900-2200	[5]
•	Carbon dioxide	60	0.80	0.2	150-850	1000-1200	[5]
Ó	Carbon dioxide	73	0.97	0.1	40-60	1900	[8] [8]
-⊙		63	0.84	0.1	100-400	1600	[8]
·⊚		59	0.79	0.1	65–520	1400-1800	[8]
lacktriangle		1.03	0.0046	0.1-0.7	1000	600-1500	[9]
Ō	Water	1.03	0.0046	0.20	800-1000	600-700	[9] [7]
\bullet	water	1.03	0.0046	1.00	500-800	400-500	[7]
		1.03	0.0046	13.0	200-500	200	[5]
		1.03	0.030	4.78	300-500	210-240	[1]
		1.03	0.030	6.05	130-530	190-220	וֹן זֹן
	n-pentane	1.03	0.030	8.94	130-530	150-200	[ו]
		1.03	0.030	13.0	100-400	200	[5]
×	Hexane	1.03	0.033	13.0	200-400	200-240	[5]
+	Ethyl alcohol	1.03	0.016	8.94	140-540	170-190	[1]
×	Emyr alconor	1.03	0.016	13.0	200-400	220	[5]
\$	Carbon	1.03	0.022	1.0	450-850	230-370	[7]
*	tetrachloride	1.03	0.022	8.94	450-850	100	[וֹ]
Δ		1.03	0.019	6:35	80–160	200-220	[2]
$\overline{m{\Delta}}$	Iso-propanol	1.03	0.019	17:1	80–150	170	[2]
A		1.03	0.019	48.1	90–150	170	$\begin{bmatrix} 2 \end{bmatrix}$
	-	34.5	0.88	0.1	50-110	1100	[8]
	Freon 13	29.9	0.76	0.1	120–290	1100	[8]
\otimes	F	40–49	0.79-0.97	0.05	40-400	1500-1900	[5]
⊗ ×	Freon 22	40-49	0.79-0.97	0.5	30–400	800–900	[5]
•		1.03	0.030	4.7	170	150	[2]
(Freon 113	1.03	0.030	8.0	100-170	120	[2]
\Rightarrow	11con 113	1.03	0.030	48.1	100-160	120	[2]
\Diamond		1.03	0.030	4.76	150-250	130-260	[10]

the diameter of the cylinder is not sufficiently large compared to the film thickness, we cannot assume a boundary-layer treatment for such a situation.

By using the parameters adopted in the presentation of Fig. 3, many experimental results over a wide range of system parameters are plotted in Fig. 4. These plots involve also the data in Fig. 3. The explanation of the keys in the figure is tabulated in Table 3. The diameter of cylinder ranges from about 0.006 to about 48 mm, the system pressure from atmospheric to critical pressure and twelve kinds of liquid are included. In spite of the wide range of system parameters the correlation is fairly good. The solid straight line labelled with "a" in Fig. 4 corresponds to the equation (30).

For the value of abscissa less than about 10⁵ the experimental results deviate from the curve "a" considerably. This may be attributed to the same reason as that discussed above. The broken straight line labelled with "b" is a faired one which is drawn so as to fit the data having a abscissa less than 10⁵. It is to be written as.

$$\overline{Nu}_D = 1.82 \left[Gr_D \cdot \frac{Pr^{*2}}{Pr^* + 1.33} \right]^{\frac{1}{6}}.$$
 (32)

A curve can be drawn which fairs approximately all the experimental data. This curve marked by "c" is represented as follows.

$$Y = 0.22 + 0.15 X + 0.0058 X^2$$
 (33)

$$X \equiv \log_{10} \left[Gr_D \frac{P_r^{*2}}{P_r^{*} + 1.33} \right]$$
 (34)

$$Y \equiv \log_{10} \left[\overline{Nu}_{\rm D} \right]. \tag{35}$$

Considering the fact that the dimensionless parameters adopted as the basis of the correlation were derived from the laminar boundary-layer treatment of the vapour film, it is somewhat beyond expectation that these parameters still possess some meaning for the correlation of film boiling data where the concept of boundary layer seems to be inapplicable. This is a promising clue for the analysis of film boiling in which

a thin wire or a large superheating is considered.

On the other hand the Frederking's method of correlation [3],

$$\overline{Nu}_{D} \text{ vs. } Gr_{D} \left[\frac{Pr_{F}^{*2}/(Pr_{F}^{*} + 1.143)}{Pr^{2}/(Pr + 1.143)} \right]
Pr_{F}^{*} \equiv Pr \left(1 + 2.5 \frac{r_{v}}{Cp_{v} \Delta T_{\text{sat}}} \right)$$
(36)

can afford as good a correlation as Fig. 4. But the authors' method is said to be superior to the Frederking's in the two points: (1) the former has a more clear physical reasoning than the latter, and (2) the abscissa in the former is simpler than that in the latter.

Breen and Westwater [2] extended the correlation of Bromley [1] to a wider range of the diameter of cylinder by introducing the critical wave length of Taylor instability. However, it was found by authors' examination that their method failed to correlate the data of film boiling of carbon dioxide [5] and helium [3] from thin wire. An extension of correlation by the critical wave length which is determined independently of the superheating and the geometry of the heat transferring surface might result in a limited success.

4. CONCLUSION

Film boiling heat transfer from a horizontal cylinder to a stagnant saturated liquid was analysed by the integral method. A theoretical correlating equation (30) was derived in the region where the laminar boundary-layer treatment seemed to be applicable. By making use of the same dimensionless parameters as those appearing in the equation, a semi-theoretical correlating equation (33) was determined, which compared favourably with all the data available. Film boiling heat transfer of saturated liquid from a horizontal cylinder can be predicted with sufficient accuracy by this semi-theoretical equation.

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TRANSFERT THERMIQUE PAR UN CYLINDRE HORIZONTAL À DES LIQUIDES SATURÉS LORS D'UNE ÉBULLITION EN RÉSERVOIR AVEC FILM

Résumé—Il a été proposé de nombreuses formules de transfert thermique se rapportant au transfert thermique à convection naturelle lors de l'ébullition en film. Cependant aucune d'elles n'unifie les résultats utilisables pour une large variété de liquide bouillant, de diamètre du cylindre, de degré de surchauffe et de pression du système.

On applique la méthode intégrale de la couche limite pour résoudre l'équation de quantité de mouvement et celle d'énergie pour ce problème, en tenant compte de la force d'inertie dans la première et du terme de convection dans la seconde et on déduit de cette analyse deux paramètres adimensionnels.

En employant ces paramètres, on propose une équation semi-théorique assez précise pour la pratique dans un large domaine de variation des paramètres.

WÄRMEÜBERGANG BEI FILMSIEDEN IM BEHÄLTER, VON EINEM WAAGRECHTEN ROHR AN FLÜSSIGKEITEN IM SÄTTIGUNGSZUSTAND

Zusammenfassung—Für den Wärmeübergang beim Filmsieden in freier Konvektion an waagerechten Rohren wurden viele verschiedene Korrelationsbeziehungen vorgeschlagen. Doch keine davon liefert eine gemeinsame Beziehung für die verfügbaren Daten über einen weiten Bereich der Flüssigkeitseigenschaften, des Zylinderdurchmessers, des Überhitzungsgrades und des Druckes im System.

Es wurde die integrale Grenzschichtbetrachtung angewandt, um die Kräfte- und Energiegleichungen für dieses Problem zu lösen. Dabei wurde für die erste Beziehung die Massenträgheit, für die zweite der konvektive Anteil in Betracht gezogen. Zwei dimensionslose Parameter wurden aus der Analyse gewonnen. Mit Hilfe dieser Parameter ergab sich eine halbtheoretische Beziehung, die sich für die Praxis über einen weiten Bereich von Grössen als genügend genau erwies.

ПЕРЕНОС ТЕПЛА ОТ ГОРИЗОНТАЛЬНОГО ЦИЛИНДРА К НАСЫЩЕННОЙ ЖИДКОСТИ ПРИ ПЛЕНОЧНОМ КИПЕНИИ В БОЛЬШОМ ОБЪЕМЕ

Аннотация—Предложено много типов зависимостей, характеризующих теплообмен при пленочном кипении в условиях свободной конвекции. Однако ни одна из этих зависимостей непригодна для обобщения имеющихся данных в широком диапазоне изменения диаметров цилиндров и давления в системе кипящих жидкостей различного рода.

Для решения уравнений пограничного слоя в этой задаче применялся интегральный метод, при чем в уравнении движения учитывалась сила инерции, а в уравнении энергии конвективный член. Из этого анализа выведены безразмерные параметры. В результате использования этих параметров предложено полуэмпирическое обобщающее уравнение, достаточно точное для практических целей в широком диапазоне параметров.